# Mathematics: Applications & Interpretation

# SL & HL

# Formula Sheet

## Topic 1: Prior Learning SL & HL

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|---|---|
| Area: Parallelogram | $A = bh,\ b = base,\ h = height$ |
| Area: Triangle | $A = \frac{1}{2}(bh),\ b = base,\ h = height$ |
| Area: Trapezoid | $A = \frac{1}{2}(a + b)h,\ a, b = parallel\ sides,\ h = height$ |
| Area: Circle | $A = \pi r^2,\ r = \text{radius}$ |
| Circumference: Circle | $C = 2\pi r,\ r = radius$ |
| Volume: Cuboid | $V = lwh,\ l = length,\ w = width,\ h = height$ |
| Volume: Cylinder | $V = \pi r^2 h,\ r = \text{radius},\ h = \text{height}$ |
| Volume: Prism | $V = Ah,\ A = cross-section\ area,\ h = height$ |
| Area: Cylinder curve | $A = 2\pi rh,\ r = \text{radius},\ h = \text{height}$ |
| Distance between two points $(x_1, y_1), (x_2, y_2)$ | $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ |
| Coordinates of midpoint | $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right),\ for\ endpoints\ (x_1, y_1), (x_2, y_2)$ |

## Topic 1: Prior Learning HL Only

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| Solutions of a quadratic equation in the form $ax^2 + bx + c = 0$ | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a \neq 0$ |

# Topic 2: Number and Algebra SL & HL

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| The nth term of an arithmetic sequence | $u_n = u_1 + (n-1)d$ |
| Sum of n terms of an arithmetic sequence | $S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right) = \frac{n}{2}\left(u_1 + u_n\right)$ |
| The nth term of a geometric sequence | $u_n = u_1 r^{n-1}$ |
| Sum of n terms of a finite geometric sequence | $S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}, \quad r \neq 1$ |
| Compound Interest | $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$<br>FV is future value, PV is present value, n is the number of years, k is the number of compounding periods per year, r% is the nominal annual rate of interest |
| Exponents and Logarithms | $a^x = b \Leftrightarrow x = \log_a b, \quad a, b > 0, \quad a \neq 1$ |
| Percentage error | $\varepsilon = \left\lvert \frac{V_A - V_E}{V_E} \right\rvert \times 100\%$<br>$V_A = approximate\ value,\ V_E = exact\ value$ |

# Topic 2: Number and Algebra HL only

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| Laws of logarithms for a, x, y > 0 | $\log_a xy = \log_a x + \log_a y$<br>$\log_a \frac{x}{y} = \log_a x - \log_a y$<br>$\log_a x^m = m \log_a x$ |

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| The sum of an infinite geometric sequence | $S_\infty = \frac{u_1}{1-r}, \ \lvert r\rvert < 1$ |
| Complex numbers | $z = a + bi$ |
| Discriminant | $\triangle = b^2 - 4ac$ |
| Modulus - argument (polar) and Exponential (Eular) form | $z = r(\cos\theta + i\sin\theta) = re^{i\theta} = rcis\theta$ |
| Determinant of a 2 × 2 matrix | $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow det\ A = \lvert A\rvert = ad - bc$ |
| Inverse of a 2 × 2 matrix | $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{det\ A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ |
| Power formula for a matrix | $M^n = PD^nP^{-1}$<br>Where P is the matrix of eigenvectors and D is the diagonal matrix of eigenvalues |

# Topic 3: Functions SL & HL

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| Equations of a straight line | $y = mx + c\ ;\ ax + by + d = 0;$<br>$y - y_1 = m(x - x_1)$ |
| Gradient formula | $m = \frac{y_2 - y_1}{x_2 - x_1}$ |
| Axis of symmetry of a quadratic function | $f(x) = ax^2 + bx + c \Rightarrow x = -\frac{b}{2a}$ |

# Topic 3: Functions HL only

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| Logistic function | $f(x) = \frac{L}{1 + Ce^{-kx}}, \ L, k, C > 0$ |

# Topic 4: Geometry and Trigonometry SL & HL

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| Distance between two points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ | $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ |
| Coordinates of midpoint of a line with endpoints $(x_1, y_1, z_1), (x_2, y_2, z_2)$ | $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$ |
| Volume: Right - pyramid | $V = \frac{1}{3}Ah, \ A = base\ area, \ h = height$ |
| Volume: Right cone | $V = \frac{1}{3}\pi r^2 h, \ r = \text{radius}, \ h = \text{height}$ |
| Area: Cone curve | $A = \pi r l, \ r = \text{radius}, \ l = \text{slant height}$ |
| Volume: Sphere | $V = \frac{4}{3}\pi r^3, \ r = \text{radius}$ |
| Surface area: Sphere | $A = 4\pi r^2, \ r = \text{radius}$ |
| Sine rule | $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ |
| Cosine rule | $c^2 = a^2 + b^2 - 2ab\cos C$<br>$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ |
| Area: Triangle | $A = \frac{1}{2}ab\sin C$ |
| Length of an arc | $l = \frac{\theta}{360} \times 2\pi r$<br>$\theta = \text{angle in degrees}, \ r = \text{radius}$ |
| Area of a sector | $A = \frac{\theta}{360} \times \pi r^2$<br>$\theta = \text{angle in degrees}, \ r = \text{radius}$ |

# Topic 4: Geometry and Trigonometry HL only

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| Length of an arc | $l = r\theta$<br>$r = radius, \ \theta = angle\ in\ radians$ |

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| Area of a sector | $A = \frac{1}{2} r^2\theta$<br>$r = radius,\ \theta = angle\ in\ radians$ |
| Identities | $\cos^2\theta + \sin^2\theta = 1$<br>$\tan\theta = \frac{\sin\theta}{\cos\theta}$ |
| Transformation matrices | $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$<br>. *reflection in the line* $y = (\tan\theta)\,x$<br><br>$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$<br>. *horizontal stretch by scale factor of* $k$<br><br>$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$<br>. *vertical stretch with scale factor of* $k$<br><br>$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$, *centre* $(0, 0)$<br>. *enlargement with scale factor of* $k$<br><br>$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$<br>. *anticlockwise rotation of angle* $\theta$ *about the origin* $(\theta > 0)$<br><br>$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$<br>. *clockwise rotation of angle* $\theta$ *about the origin* $(\theta > 0)$ |

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| Magnitude of a vector | $\lvert v\rvert = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2}$ |
| Vector equation of a line | $r = a + \lambda b$ |
| Parametric form of the equation of a line | $x = x_0 + \lambda l,\ y = y_0 + \lambda m,\ z = z_0 + \lambda n$ |
| Scalar product | $v \cdot w = v_1 w_1 + v_2 w_2 + v_3 w_3$<br>$v \cdot w = \lvert v\rvert\lvert w\rvert \cos\theta$<br>*where $\theta$ is the angle between v and w* |
| Angle between two vectors | $\cos\theta = \dfrac{v_1 w_1 + v_2 w_2 + v_3 w_3}{\lvert v\rvert\lvert w\rvert}$ |
| Vector product | $v \times w = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix}$<br>$\lvert v \times w\rvert = \lvert v\rvert\lvert w\rvert \sin\theta$<br>*where $\theta$ is the angle between v and w* |
| Area of a parallelogram | $A = \lvert v \times w\rvert$<br>Where v and w form two adjacent sides of a parallelogram |

# Topic 5: Statistics and Probability SL & HL

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| Interquartile range | $IQR = Q_3 - Q_1$ |
| Mean, $\bar{x}$ , of a set of data | $\bar{x} = \dfrac{\sum_{i=1}^{k} f_i x_i}{n}$ , *where* $n = \sum_{i=1}^{k} f_i$ |
| Probability of an event A | $P(A) = \dfrac{n(A)}{n(u)}$ |
| Complementary events | $P(A) + P(A') = 1$ |
| Combined events | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ |
| Mutually exclusive events | $P(A \cup B) = P(A) + P(B)$ |

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| Conditional probability | $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ |
| Independent events | $P(A \cap B) = P(A)\,P(B)$ |
| Expected value of a Discrete random variable x | $E(X) = \sum x\,P(X = x)$ |
| Binomial distribution Mean; Variance | $X \sim B(n, p)$<br>$E(X) = np;$<br>$Var(X) = np(1-p)$ |

# Topic 5: Statistics and Probability HL only

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| Linear transformation of a single random variable | $E(aX + b) = aE(X) + b$<br>$Var(aX + b) = a^2 Var(X)$ |
| Linear combinations of n independent random variables, $X_1, X_2, \ldots\ldots X_n$ | $E(a_1X_1 \pm a_2X_2 \pm \ldots. \pm a_nX_n) =$<br>$a_1E(X_1) \pm a_2E(X_2) \pm \ldots.. \pm a_nE(X_n)$<br>$Var(a_1X_1 \pm a_2X_2 \pm \ldots. \pm a_nX_n) =$<br>$a_1^2Var(X_1) + a_2^2Var(X_2) + \ldots.. + a_n^2Var(X_n)$ |
| Unbiased estimate of population variance | $S_{n-1}^2 = \frac{n}{n-1} S_n^2$ *Sample statistics* |
| Poisson distribution Mean; Variance | $X \sim Po(m)$<br>$E(X) = m\,;\, Var(X) = m$ |
| Transition matrices | $T^n s_0 = s_n$<br>*where* $s_0$ *is the initial state* |

# Topic 6: Calculus SL & HL

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| Derivative of $x^n$ | $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$ |
| Integral of $x^n$ | $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ |

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| Area enclosed by a curve and the x - axis | $A = \int_a^b y\,dx, \quad where\ f(x) > 0$ |
| The trapezoidal rule where $h = \frac{b-a}{n}$ | $\int_a^b y\,dx \approx \frac{1}{2}h\left(\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + \ldots\ldots + y_{n-1}\right)\right)$ |

# Topic 6: Calculus HL only

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| Derivative of sin x | $f(x) = \sin x \Rightarrow f'(x) = \cos x$ |
| Derivative of cos x | $f(x) = \cos x \Rightarrow f'(x) = -\sin x$ |
| Derivative of tan x | $f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$ |
| Derivative of $e^x$ | $f(x) = e^x \Rightarrow f'(x) = e^x$ |
| Derivative of ln x | $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$ |
| Chain rule | $y = g(u),\ u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ |
| Product rule | $y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ |
| Quotient rule | $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ |
| Standard integrals | $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$<br>$\int \sin x\,dx = -\cos x + C$<br>$\int \cos x\,dx = \sin x + C$<br>$\int \frac{1}{\cos^2 x}\,dx = \tan x + C$<br>$\int e^x\,dx = e^x + C$ |

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| Area enclosed by a curve and x or y - axes | $A = \int_a^b \lvert y \rvert \, dx \qquad or \qquad A = \int_a^b \lvert x \rvert \, dy$ |
| Volume of revolution about x or y - axes | $V = \int_a^b \pi y^2 \, \mathrm{d}x \qquad or \qquad V = \int_a^b \pi x^2 \, \mathrm{d}y$ |
| Acceleration | $a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = v\frac{dv}{ds}$ |
| Distance; Displacement traveled from $t_1$ to $t_2$ | $dist = \int_{t_1}^{t_2} \lvert v(t) \rvert \, dt \;\; ; \; disp = \int_{t_1}^{t_2} v(t) \, \mathrm{d}t$ |
| Euler's method | $y_{n+1} = y_n + h \times f\left(x_n, y_n\right);$<br>$x_{n+1} = x_n + h$<br>Where h is a constant (step length) |
| Euler's method for coupled systems | $x_{n+1} = x_n + h \times f_1\left(x_n, y_n, t_n\right)$<br>$y_{n+1} = y_n + h \times f_2\left(x_n, y_n, t_n\right)$<br>$t_{n+1} = t_n + h$<br>Where h is a constant (step length) |
| Exact solution for coupled linear differential equations | $x = Ae^{\lambda_1 t} p_1 + Be^{\lambda_2 t} p_2$ |